Vacuum-Air Missile Boost System

SUDHIR KUMAR*
Army Research Office-Durham, Durham, N. C.

JAI R. N. RAJAN†
Duke University, Durham, N. C.

AND

JAMES J. MURRAY‡
Army Research Office-Durham, Durham, N. C.

The vacuum-air missile boost system consists of a partially evacuated vertical launching tube with a breakable seal at the top and with the lower end sealed by the missile on a held-down sabot. When the sabot is released, atmospheric pressure from air entering the base of the tube accelerates the missile. A "muzzle chamber" near the top of the tube reduces the velocity loss due to compression of the residual air above it, and the tube length beyond the muzzle chamber serves as an "atmospheric shock reducer." The system is considered suitable for small- and medium-sized missiles. Advantages compared to the usual surface-launching technique are a significant fuel saving or an increase in payload, less sensitivity to surface gusts, and a shelter prior to firing. The motion of the missile is analyzed by assuming incompressible, quasi-steady air flow; effects of muzzle chamber size, viscous loss, and turbulence are included. Theoretical curves of muzzle velocity vs vehicle weight are given for various tube lengths for maximum accelerations of 8 and 15 g's. Muzzle velocities of 600 to 800 fps appear to be feasible at these acceleration levels. Results obtained in experiments in 3- and 1.25-in.-i.d. by 20-ft tubes show good agreement with theory.

Nomenclature

A = area of cross section of the tube
a = $f\bar{L}$
b = $\bar{m}f\bar{L} - 1$
D = diameter of cross section of the tube
d = $2/m\gamma T$
f = Darcey's friction coefficient for the given tube
G = number of g acceleration initially given to the missile
g = acceleration due to gravity
h = $v^2/2gL$
j = $T\bar{m}(1 - \bar{p}_i)$
k = $(1 + (1/\bar{m}))$
L = length of tube from $x = 0$ to effective center of chamber
$\bar{L}$ = L/D
m = mass of missile and carriage
$\bar{m}$ = $m/\rho AL$, characteristic scaling parameter of system
n = volume divided by rate of evacuation of vacuum pumps
P = prevailing atmospheric pressure
p = air pressure
$\bar{p}_i$ = p_{ie}/P
p_{ie} = $(p_i/2)\{1 + [1 + (V/V_c)]^\gamma\}$
p_i, p_{ie}, p_T = initial, effective, and terminal internal pressures
q = $a\bar{m}$
r = internal radius of cross section of tube
Re = Reynolds number of flow inside tube
s = coefficient of solid friction between tube and missile carriage
T = $PA/mg = G + 1$, or $A(P - p_{ie})$ in Eq. (5)
V, V_c = volume and volume of muzzle chamber
$\bar{v}$ = $v(\rho/\gamma P)^{1/2}$ = ratio of velocity to velocity of sound in atmosphere
v, v_T = velocity and terminal (at muzzle end) velocity
x, x_T = distance along tube starting from initial missile position; x_T corresponds to achievement of v_T
γ = ratio of specific heats of air
ϵ = length representative of absolute roughness of tube surface
ν = coefficient of kinematic viscosity
ρ = density of air under prevailing atmospheric conditions

Received August 29, 1963; revision received April 6, 1964. This work was conducted in the Department of Civil Engineering, Duke University, under the sponsorship of the Army Research Office-Durham. The present paper is based on the detailed Interim Technical Rept. No. 3, Project Mountainwell, Army Research Office-Durham, September 1962. The authors are indebted to G. L. Dugger for his constructive efforts in improving the manuscript of this paper.

* Associate Director, Engineering Sciences Division, and Visiting Associate Professor, Department of Civil Engineering, Duke University. Member AIAA.

† Research Engineer, Department of Civil Engineering.

‡ Director, Engineering Sciences Division.

Introduction

THE amount of propellant required to lift a missile from a launch pad and impart to it the first 500–1000 fps velocity is quite high—as much as 20–40% of its total weight. It has been suggested[1] that most of this propellant could be saved by launching a missile through an evacuated tube, using atmospheric pressure behind it as the driving force. For applications where a large number of vehicles are to be launched from a given site and where moderate initial acceleration (8 to 15 g) is acceptable (or maybe even desirable), such a vacuum-air boost system would reduce system operating costs and offer the side advantages of silo operation (prelaunch protection) and initial launch velocity (reduced surface-wind sensitivity and more effective aerodynamic steering, if any). Compressed-gas systems are, of course, already in use [Polaris,[2] Nike,[3] and Mobile Medium Range Ballistic Missile (MMRBM[4])]. They permit use of a shorter tube for military applications where greater acceleration and/or launch-system mobility are either permissible or necessary, but the present system could be advantageous for meteorological or small space-launching systems.

A schematic diagram of the suggested arrangement is given in Fig. 1. The same tube could be used for launching missiles of various sizes by varying the degree of evacuation and the weight of the sabot in order to keep the maximum

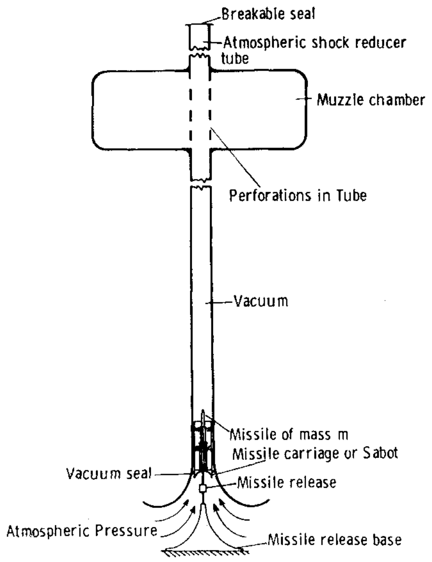

Fig. 1 Schematic diagram of the vacuum-air boost.

acceleration within desired limits. The sabot is equipped with a vacuum seal and a release mechanism. As the accelerating sabot and missile approach the muzzle end, the top seal is quickly opened, either by mechanical means or by rapidly burning it. For the latter combustible type of seal, an aluminum sandwich shell[5] containing either an explosive or an oxidizer such as ammonium perchlorate might be used; the resulting flame would be displaced by the air pushed ahead of the missile, thus enabling the missile to exit safely. The large "muzzle chamber" near the top of the tube (not incorporated in an earlier arrangement[1]) reduces the velocity loss that would otherwise result from the compression of the low-pressure air in the "evacuated" tube (e.g., for an initial pressure of 0.5 psi, the velocity loss could be 10–20% for $L/D = 150$). Preliminary studies indicate that the volume of the chamber should be greater than half the volume of the tube, and the total area of the openings between the tube and the muzzle chamber should be larger than the cross-sectional area of the tube.

When the seal is opened, the atmospheric air rushes in and impacts the missile with a shock. This effect can be mitigated by extending the tube beyond the muzzle chamber. The length of this "shock-reducer" extension will depend on the evacuation pressure and the volumes of the muzzle chamber and the tube. The seal should be opened when the pressure in the shock-reducer section reaches (or slightly exceeds) one atmosphere. The negative impulse thus imparted to the missile is proportional to the volume of the shock-reducer section and inversely proportional to the kinetic energy of the missile. It is small (1% v_T for shock-reducer length $= L/20$) in comparison with the gain imparted by the muzzle chamber (10–25% v_T), and since the size and usage of the shock-reducer tube would depend on many engineering design considerations, its influence will not be considered here in the theoretical analysis.

Because very large volumes of air have to be evacuated in the present system, large capacity vacuum pumps are required (see Appendix). For example, to produce 1 psia in a 30-ft-diam by 2000-ft tube with a muzzle chamber of about the same volume, ten pumps of 25,000 ft^3/min capacity would take 27 min.

The maximum acceleration imparted to the missile can be readily controlled; it is proposed here to hold it to 8 g's. For rugged, solid propellant rockets, acceleration up to 50 g's might be used.

Among the advantages of this system, the most significant is an appreciable saving (up to 50%)[6] in fuel and associated structural weight for delivery of a given payload to a certain altitude, range, or orbit. Alternately, it can offer up to 30% increase in payload.[6] The initial velocity also makes the missile less sensitive to low-altitude turbulence or gusts, and aerodynamic stabilizers become more effective, if used. The shelter provided by the tube has obvious advantages; for example, the equipment can be maintained at uniform temperature during the checkout phase.

The authors realize that the design and construction problems would be appreciable and that there would be problems in modifying any existing missile for use of this principle. However, the problems are well within our present technological capability, and this boosting system is believed to warrant serious consideration for use with small- and medium-sized missiles. Because of the relatively small tubes required, small space missiles (such as Scout or Astrobee),[7] upper atmosphere research rockets (such as Aerobee or Nike-Cajun),[8] and some meteorological sounding rockets would be candidates for initial development.

Theoretical Analysis

The following assumptions were made for most of the present analysis§:

1) The air which flows into the base of the tube is incompressible.

2) The air left within the evacuated section of the tube is compressed adiabatically by the missile and sabot.

3) The flow is quasi-steady. In reality the flow in the tube is unsteady, but this simplifying assumption, which facilitates the viscous friction loss estimation, is considered a good approximation for velocities up to 600–800 fps.

4) The friction between the sabot and the tube is taken as Coulomb friction, i.e., it is a constant multiple of the weight of the projectile and does not depend on its velocity.

5) The seal at the muzzle end acts as open when the internal pressure exerts a force on the seal equal to the sum of the forces exerted by the atmospheric pressure and the bond strength between the seal and the tube.

6) Leakage of air around the projectile into the evacuated part of the tube during the launching can be neglected.

The internal pressure p_i could be estimated at each instant by assuming adiabatic compression of the unevacuated air, but this would make the equation of motion nonlinear. If the muzzle chamber is comparable to the tube volume, the change in p_i is rather small and the problem may be linearized by assuming a constant effective internal pressure p_{ie}, which is a mean of p_i and p_T. Thus

$$p_{ie} = (p_i/2)\{1 + [1 + (V/V_c)^\gamma]\} \tag{1}$$

Since the Reynolds numbers exceed 2000 for most of the flight of the missile in the tube, transitional and completely turbulent flows are expected. Qualitative flow diagrams for these two types of flows and a suggested shape of the sabot are given in Fig. 2. For the transitional flow (Fig. 2a), the turbulent boundary layer starts very near the entrance, and its thickness increases with distance but never fills the tube completely. In the completely turbulent flow case (Fig. 2b), the flow is turbulent everywhere except near the breach end and immediately behind the missile. For both of the foregoing cases we may apply as an approximation the well-known Blasius-Darcey-Moody equation for the loss of pressure head due to friction:

$$h_f = f(x/D)(\rho v^2/2) \tag{2}$$

The dimensionless friction factor f is a function of the relative roughness, ϵ/D; for the transitional case it also varies with the Reynolds number Re.[9] Table 1 gives estimates

§ Effects of viscosity losses, energy lost in shattering or opening the seal in the muzzle, will be discussed during the solution.

of these parameters for tubes from 1 in. to 80 ft in diameter. For commercial steel or wrought iron tubes, the flow never becomes completely turbulent, but for smooth concrete it will be turbulent for velocities higher than 200 or 300 fps. For concrete tubes of 10- to 80-ft in diameter with turbulent flow, f varies only from 0.012 to 0.008, with 0.01 as an approximate mean value.

When the flow is transitional or a combination of transitional and completely turbulent, f varies with Re, changing as the missile accelerates in the tube. Under these conditions f is given by[10]

$$f^{-1/2} = -2 \log \left(\frac{\epsilon/D}{3.7} + \frac{2.51}{Re\, f^{1/2}}\right) \tag{3}$$

Hence

$$\Delta p = \int_0^L f(\rho\gamma^2)/2D.dx \tag{4}$$

which can be computed numerically for a given problem. However, a good approximation can be obtained by simply using an effective f in Eq. (2). This will be discussed later in the solution for the transitional flow case.

Completely Turbulent Flow in Tube with Muzzle Chamber

As has been noted, f is a constant for a given ϵ/D. The thrust due to the pressure differential at the two ends of the missile (Fig. 1) is

$$T = A(P - p_{ie}) \tag{5}$$

where p_{ie} is given by Eq. (1); hence, the equation of motion

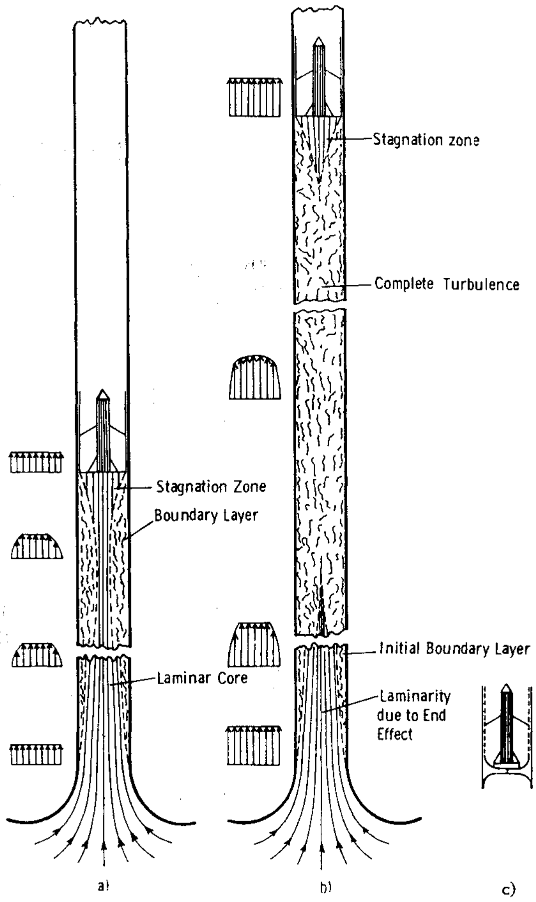

Fig. 2 Qualitative flow diagrams: a) transitional, b) turbulent, and c) sabot.

Table 1 Flow parameters for various launching tubes for atmospheric air at 60°F

Tube diam, ft	Tube surface	$\epsilon/D \times 10^6$	$vD\ x\ 10^{-2}$, ft²/sec	$Re^a \times 10^{-6}$	$f^b \times 10^3$
0.08–1.0	Drawn tubing	60–5	125	100	44–10
1.0–10	Steel or wrought iron	150–15	17 125	10 100	45–14
	Smooth concrete	1000–120	2 29	1 18	45–20 44–12
10–30	Steel or wrought iron	15–5	167 167	300 1000	≤7.5 ≤7.5
	Smooth concrete	100–40	29 67	18 50	45–12 42–10
	Smooth riveted steel	300–100	58 29	5 18	45–16 45–12
30–80	Smooth concrete	40–15	67 167	50 180	45–10 44–8

[a] For transitional flow, these are maximum values, because Re can be as low as 3000 for any case. Flow is turbulent for listed Re and larger values.
[b] The higher f corresponds to Re = 3000 for transitional flow; the lower corresponds to the listed Re and larger values.

of the missile, carriage, and the column of air behind it in the tube may be written

$$\frac{1}{2}(m + \rho Ax)\frac{dv^2}{dx} + \frac{1}{2}A\rho\left(f\frac{x}{d} + 1\right)v^2 + A\rho xg + mg(1 + s) - A(P - p_{ie}) = 0 \tag{6}$$

An equation similar to this was recently solved numerically for one case.[11]

Equation (6) may be nondimensionalized in several ways. Two useful ways are

$$v^2/a^2 = \phi(x/L, L/D, m/\rho AL, PA/mg, p_{ie}/P, \gamma) \tag{7a}$$

$$v^2/2Lg = \psi(x/L, L/D, m/\rho AL, PA/mg, p_{ie}/P) \tag{7b}$$

where $\bar{m} = m/\rho AL$ is a characteristic scaling parameter of the problem (see Appendix), $v^2/2gL$ is a Froude number of the problem which gives a nondimensional height to which the missile would go vertically against gravity inertially, air drag being neglected.

The equation of motion may thus be expressed as

$$\frac{d\bar{v}^2}{d\bar{x}} + \left(a - \frac{b}{\bar{m} + \bar{x}}\right)\bar{v}^2 = d\left(\frac{j}{\bar{m} + \bar{x}} - 1\right) \tag{7}$$

or

$$\frac{dh}{d\bar{x}} + \left(a - \frac{b}{\bar{m} + \bar{x}}\right)h = \left(\frac{j}{\bar{m} + \bar{x}} - 1\right) \tag{8}$$

Equations (7) and (8) are identical except for the constant d, which is unity in the second and not in the first. With this change, solution of one would apply to the other. Using the initial conditions for the problem as $v = h = 0$ at $x = 0$, solution of Eq. (7) is given by

$$\bar{v}^2 = e^{-a\bar{x}}\,\frac{(\bar{m} + \bar{x})^b}{\bar{m}^b}\cdot d\left[\left(\frac{ja}{b} - 1\right)\int_0^{\bar{x}} \frac{e^{at}}{[1 + (t/\bar{m})]^b}\cdot dt = \frac{j}{b}\left\{\frac{e^{a\bar{x}}}{[1 + (\bar{x}/\bar{m})]^b} - 1\right\}\right] \tag{9}$$

The nondimensional terminal velocity $\bar{v}_T$ is then given by

$$\bar{v}_T = e^{-a}\left(1 + \frac{1}{\bar{m}}\right)^b d\left[\left(\frac{ja}{b} - 1\right)\int_0^1 \frac{e^{at}}{(1 + (t/\bar{m}))^b}\cdot dt - \frac{j}{b}\left\{\frac{e^a}{[1 + (1/\bar{m})]^b} - 1\right\}\right] \tag{10}$$

The finite integral in Eq. (10) is evaluated by putting $x = at - am \log[1 + (t/m)]$. For the range of values of the constants a and b, with which we are mostly concerned, and with error less than a fraction of a percent, it is given as

$$\int_0^1 \frac{e^{at}}{[1 + (t/\bar{m})]^b} \cdot dt = (k + 1)\left(\frac{1}{2} - \frac{1}{4} q + \frac{1}{8} q^2\right) + (k^2 + k + 1) \frac{q}{3}\left(1 - \frac{2}{3} q\right) + (k^2 + 1)(k + 1) \frac{q^2}{8} - \frac{q\bar{m}}{12} (q + 4a + 6)k^2 \log k \quad (11)$$

where $k = 1 + (1/\bar{m})$ and $q = a\bar{m}$. The error in the integral for $\bar{m} \geqslant 3$ is smaller than 0.5%. For larger $\bar{m}$ it is much more accurate.

For the special case *where the viscous friction loss is negligible* ($f = 0$), as in the small model tested by the authors, $a = 0$, $b = -1$, and Eq. (9) becomes

$$\bar{v}^2 = [d/(\bar{m} + \bar{x})][(j - \bar{m})\bar{x} - (\bar{x}^2/2)] = \{2/[(\bar{m} + \bar{x})\cdot\gamma T]\}\{[T(1 - \bar{p}_i) - (s + 1)]\bar{x} - (\bar{x}^2/2\bar{m})\} \quad (12)$$

The terminal velocity is given by

$$\bar{v}_T^2 = \frac{2}{(\bar{m} + 1)\gamma T}\left[T(1 - \bar{p}_i) - s - \left(1 + \frac{1}{2\bar{m}}\right)\right] \quad (13)$$

For directly calculating values of v_T, nondimensional terms in Eq. (13) are expressed in their proper dimensions, giving

$$v_T^2 = \frac{L}{m + \rho A L}\{[2A(P - p_{ie}) - A\rho g L] - 2\,mg(s + 1)\} \quad (14)$$

For horizontal model firings, the terminal velocity v_{hT} is given by

$$v_{hT}^2 = [2L/(m + \rho AL)][A(P - p_{ie}) - smg] \quad (15)$$

where the difference due to gravity is

$$v_{hT}^2 - v_T^2 = gL + \{gL/[1 + (1/\bar{m})]\} \simeq 2gL \ (\bar{m} \gg 1) \quad (16)$$

which is negligible compared to v_T^2 for a model with $L = 20$ ft.

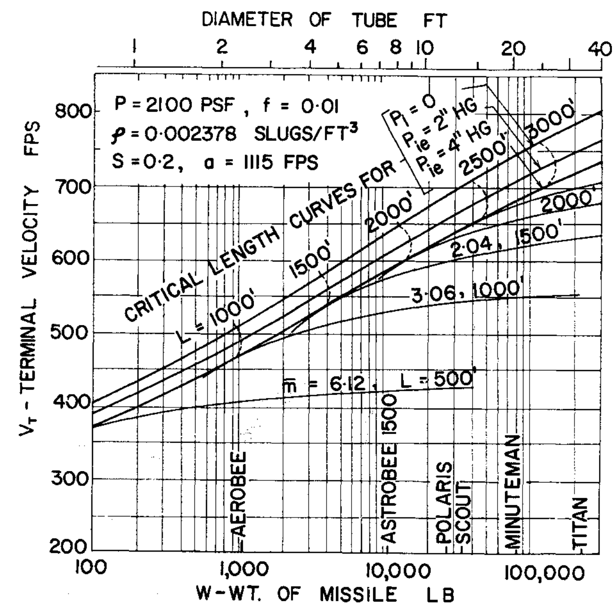

Fig. 3 Dimensional plots of v_T for p_{ie} = 4-in. Hg and G = 8, including critical length curves for p_{ie} = 0 and 2-in. Hg.

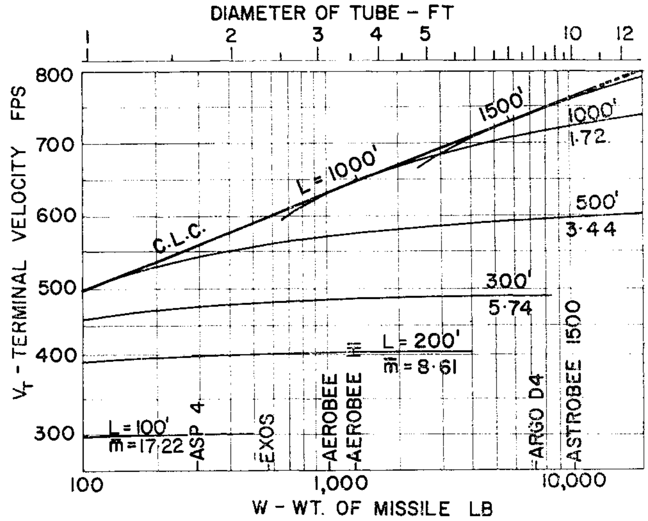

Fig. 4 Dimensional plots of v_T for p_{ie} = 0 and G = 15.

Semilog plots of v_T vs missile weight for 8-g maximum acceleration for various tube lengths are given in Fig. 3 for values of f, T, P, and s designated thereon and for $a = (P\gamma/\rho)^{1/2} = 1115$ fps. Critical-length envelope curves are given for p_{ie} = 0, 2- and 4-in. Hg along with the individual curves for p_{ie} = 4-in. Hg. The envelopes are marked at various L's; if the length is increased beyond a given L, which corresponds to the missiles $\bar{m}$ called out on the 4-in.-Hg curves, the terminal velocity of the given missile will not increase. In other words, these curves prescribe the highest velocities attainable by the present system for a given missile, if the maximum acceleration is limited to 8 g's. The terminal velocities are not affected much by p_{ie} in the range 0- to 4-in. Hg.

A scale for tube diameter, which is prescribed by the missile weight for fixed maximum acceleration, is given at the top of the graph. Weights of various missiles are indicated on the abscissa. Since the tube diameter increases with the weight of the missile and carriage, it would seem that atmospheric pressure may suffice only for launching small- and medium-size missiles, and that for missiles heavier than Minuteman (which theoretically requires a 21-ft-diam by 2800-ft tube for maximum velocity with the 8-g limit), a compressed gas system would be desirable.

Figure 4 shows the $p_i = 0$ curves for a maximum acceleration of 15 g's. This figure affords a comparative idea of the smaller lengths but larger diameters required for small- and medium-weight missiles which can stand higher accelerations. Calculations were also made for a 50-g limit, but the entire critical length curve fell above the 700 fps level, so that it could not be considered accurate, due to the assumption of incompressibility. (For the 15-g curves in Fig. 4, results for velocities above 700 fps are also of questionable accuracy). However, as an example, in a 7.3-ft-diam by 200-ft tube (less than critical length), a 2000-lb missile could theoretically reach 670 fps with a maximum acceleration of 50 g's. Thus the terminal velocities increase as the acceleration limit is raised.

Transitional Flow in Tube with Muzzle Chamber

As discussed previously, the flow in the tube will not be completely turbulent in many cases. In these cases, f varies with Re [Eq. (3)] and the differential equation of motion is nonlinear:

$$\frac{1}{2}(m + \rho Ax)\frac{dv^2}{dx} + \frac{1}{2}A\rho v^2 + \frac{1}{2}A\rho v^2 f\frac{x}{D} + A\rho gx + mg(1 + s) - A(P - p_{ie}) = 0 \quad (17)$$

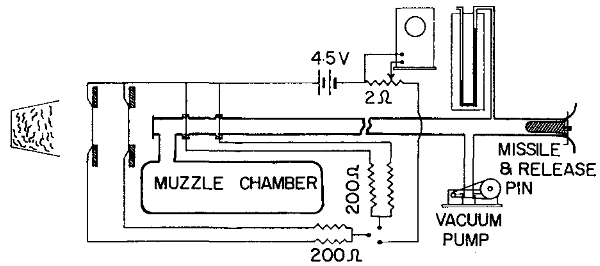

Fig. 5 Schematic arrangement of the experimental setup.

This equation may be solved numerically on a computer if considerable accuracy is desired. However, a practical solution should be possible by assuming an effective constant f for the whole tube and thus linearizing it to a form for which the solution was obtained in the previous section. As a first approximation for the effective f, its value may be taken as that which would correspond to v_T, i.e., when the missile is about to leave the muzzle end. Although the value of f is higher for smaller velocities in the transitional flow,[9] its contribution is higher for the high velocities near the muzzle end (because $\Delta p \propto v^2$), so that use of f corresponding to v_T for the whole tube is reasonable.

Tube without Muzzle Chamber

For imparting higher velocities to the missile, it is important to keep the muzzle chamber in the system; however, the present analysis is given to afford comparison of the two cases, because some of the experiments were done without the chamber. In this case, the last term in Eq. (17) is changed to $-A[P - p_i(1 - x/L)^{-2}]$. The solution to the nondimensional equation of motion is

$$\bar{v}^2 = e^{-a\bar{x}}[(\bar{m} + \bar{x})^b d \int_0^{\bar{x}} \left[\frac{T\bar{m} - \bar{m}s}{\bar{m} + t} - \frac{T\bar{m}p_i}{(\bar{m} + t)(1 + t)^\gamma} - 1 \right] \frac{e^{at}}{(\bar{m} + t)^b} \tag{18}$$

The integral would have to be computed numerically. However, the terminal velocity v_T with which the missile comes out of the tube is assumed to be acquired at a position x_T, where the force on the seal due to internal pressure becomes equal to the resultant of the other opposing forces, and if the tube is small, viscous friction losses would be negligible ($f = 0$); hence, $a = 0$ and $b = -1$, and

$$v_T^2 = \frac{2}{m + \rho A\bar{x}_T} \left(x_T(PA - mgs - mg) - x_T^2\rho Ag - \frac{p_i AL}{\gamma - 1} \left\{ \frac{1}{[1 - (x_T/L)]^{\gamma-1}} - 1 \right\} \right) \tag{19}$$

where x_T is given by

$$x_T/L = 1 - \left[\frac{p_i/P}{1 + (a/A)(1 + B)} \right]^{1/\gamma} \tag{20}$$

in which A and a are the cross-sectional areas of the tube and the face of the flange put at the muzzle end, and B is the strength of the bond between the seal and the flange. In the authors' experiments, the bond strength of silicone vacuum grease used was determined as 5.7 psi.

As discussed previously, the seal should be opened or shattered before the arrival of the missile at the muzzle, but in some cases (as in the present experiments) the missile may be allowed to shatter the seal. In such cases, it may be assumed that the sum of the kinetic energies of the missile and the seal (mass $= m_1$) after impact is equal to the kinetic energy of the missile before impact and that the velocities of the two in the axial direction after impact are the same. The velocity of the missile after emergence from the seal thus gets reduced to

$$v = v_T[m/(m + m_1)]^{1/2} \tag{21}$$

Experimental Study

Experiments were conducted with 20-ft-long copper tubes of 1.25-in. and 3-in. i.d. A schematic diagram of the experimental arrangement, which was essentially the same for the two models tested, is shown in Fig. 5. The equipment comprised the tube (and muzzle chamber, when used), a mounting system for holding the projectile and releasing it when the desired p_i was attained, and a recording oscilloscope circuit for measuring the velocities of the projectile. A

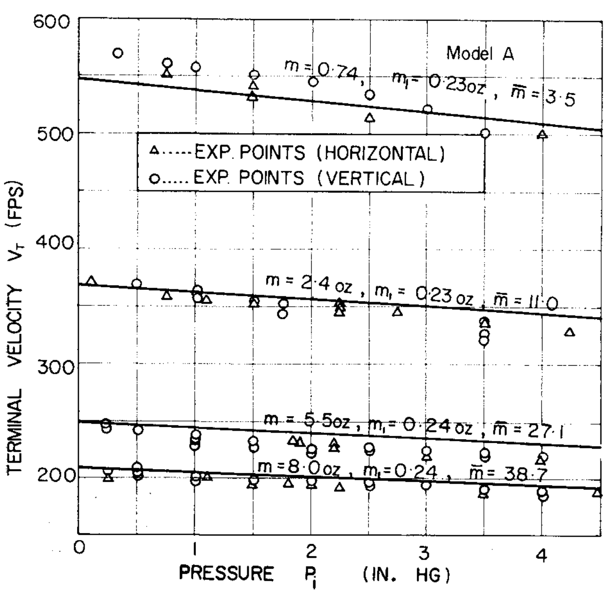

Fig. 6 Theoretical curves for horizontal firings of the 1.25-in.-i.d. model A with muzzle chamber and experimental points (velocities measured outside the tube) for both horizontal and vertical firings.

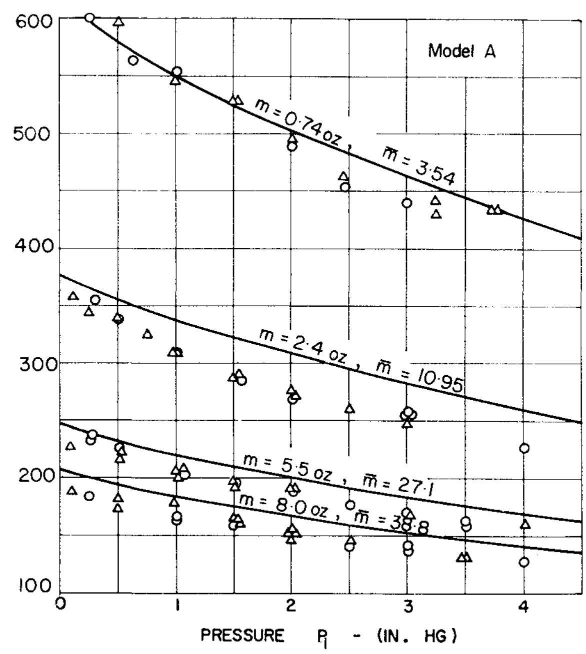

Fig. 7 Theoretical curves for horizontal firings of model A without muzzle chamber and experimental points (velocities measured inside the tube) for both horizontal and vertical firings.

target of tightly packed cotton and rags was used to stop the projectiles.

An important difference between the two models was in the manner in which the muzzle chamber was attached. In the 1.25-in. model (to be called model A), it was connected by two lines to opposite sides of the tube, whereas in the 3-in. model (to be called model B), it was integral with the tube, as shown in Fig. 1. The values of $\bar{L}$ were 190 and 80 for models A and B, respectively. The muzzle end of the tube was sealed with a thin Plexiglas plate and high-vacuum silicone grease. For model B, baffles in front of the velocity measuring foils were necessary in order to avoid their being triggered by shattered seal particles. The ratios of volumes of the muzzle chamber and the tube were 15 and 2.6 for models A and B, respectively. The projectile diameters were $\frac{1}{32}$ to $\frac{1}{16}$ in. less than the internal diameter of the tube. At the breach end, a streamlined entrance was used in model A, but its effect on the magnitude of v_T was indiscernible, and it was not provided in model B. An O-ring with an inner diameter $\frac{1}{16}$ in. smaller than the projectile and with a $\frac{1}{8}$-in. sectional diameter was found suitable for sealing the breach end. A hook was provided at the base of the projectiles to hold and release them.

The internal pressure p_i in the tube was varied from 0.5-in. Hg to 4.0-in. Hg at intervals of approximately 0.25 in., and the terminal velocities of the projectiles corresponding to different internal pressures p_i were measured. In most tests, the velocity was measured externally. The internal measurements were done only in those cases when the muzzle chamber was not attached.

The 1.22-in.-diam, Teflon projectiles tested in model A had weights of 0.74, 0.8, 1.0, 2.4, 5.5, and 8 oz. Projectiles for model B were made of seasoned, kiln-dried maple wood and weighed 3.3, 5.5, and 7.5 oz. Model A was tested in both horizontal and vertical positions, with and without the muzzle chamber (Figs. 6 and 7, respectively). Model B was tested only in the horizontal position with the muzzle chamber (Fig. 8).

The experimental results in Figs. 6–8 are compared with the theoretical curves for horizontal firings (the theoretical curves for vertical firings were so close to these that it was not necessary to show them). The following observations can be made from these figures.

1) The experimental data obtained from tests using the muzzle chamber (Figs. 6 and 8) are closer to the corresponding theoretical values than in the case without the muzzle chamber (Fig. 7).

2) The experimental terminal velocities are slightly higher than the theoretical values in some cases, for values of $\bar{m}$ less than 4. This is particularly so in the case of tests with the muzzle chamber and internal pressures p_{i_e} of less than 2 in. Hg.

3) For values of $\bar{m}$ greater than approximately 4, there is better agreement at lower values of p_{i_e} than at higher values.

4) The pressure sensitivity of the system is reduced with the introduction of the muzzle chamber as expected.

Conclusions

It is considered feasible to impart velocities up to 800 fps to light- or medium-weight rockets and missiles by the vacuum-air missile boost system. Larger velocities could be imparted to missiles that can stand higher maximum accelerations; tube lengths required for such missiles are shorter. For example, Argo D4 and Astrobee 1500 (assuming 15-g maximum acceleration) could be given almost 500 fps velocity with a tube of 10-ft i.d. and 300-ft length, in comparison with Minuteman (assuming 8-g maximum acceleration) which would require a 21-ft-i.d. by 600-ft-long tube to achieve the same velocity.

A "muzzle chamber" of volume comparable to that of the tube effects an increase of about 10–30% in terminal velocity of the missile for internal pressures of the order of 1- to 4-in. Hg.

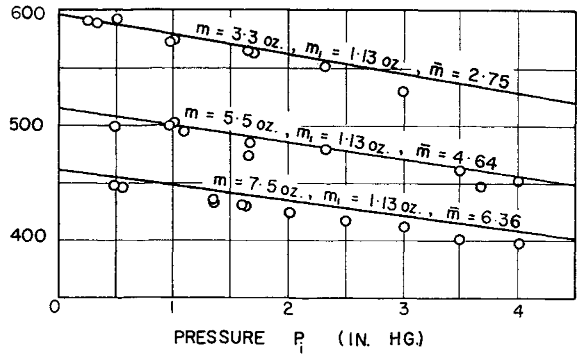

Fig. 8 Theoretical curves and experimental points for horizontal firings of the 3-in.-i.d. model B with muzzle chamber (velocities measured outside the tube).

When maximum allowable acceleration is fixed, there is a critical tube length beyond which further increase in length would not increase the velocity imparted to the missile. A reasonable practical approach probably would be one which would give about three-quarters of the maximum obtainable velocity.

Challenging engineering problems would have to be overcome before such a system becomes operational. The authors feel, however, that it is well within our present technological capability and that the savings or performance gains would be appreciable for high-launch-rate, fixed-site, small-to-medium sized missile applications.

Appendix

Time of Evacuation of Launch Tubes and Muzzle Chambers

If $S = dv/dt$ is the pumping speed of the vacuum pump and isothermal expansion of air during evacuation is assumed, then

$$dp/dt = -Sp/V$$

Further, if p_0 is the limiting pressure for the vacuum pump, and $n = V/S$, this may be written as

$$dp/dt = -(p - p_0)/n$$

which gives the time t required to evacuate volume V from P to p_i:

$$t = 2.3n[\log_{10}(P - p_0) - \log_{10}(p_i - p_0)]$$

Assuming pumping speeds of a fraction of the total volume V, the time of evacuation vs internal residual pressure p_i, as given by the foregoing equations, has been plotted in Fig. 9.

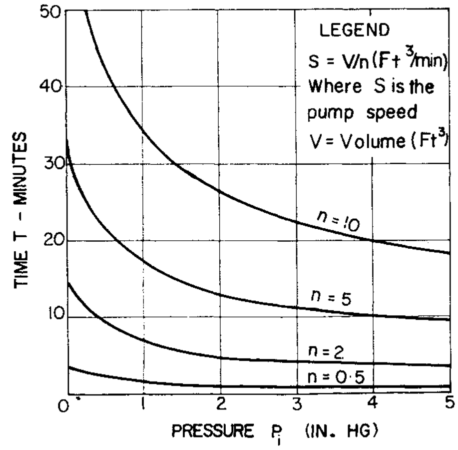

Fig. 9 Times of evacuation for various speeds.

Similitude Studies

The mass in motion in the tube, with a velocity v, may be taken as $m + \rho Ax$. The three types of forces acting on it are the inertia, friction, and gravity forces. If all three of these could be matched between model and prototype, perfect similitude would be established; however, this is not possible,[12] so it is accomplished separately by considering only inertia and friction forces in one case and inertia and gravity forces in the other.

For similitude under the action of inertia and viscous forces the important consideration is the equality of velocity between the model and prototype. The equation of motion of the missile, neglecting gravity forces, may be written as

$$(m + A\rho x)v\,\frac{dv}{d\bar{x}} = PA - \frac{1}{2}A\rho v^2 - \frac{1}{2}fA\rho v^2\,\frac{x}{D}$$

which, on using $\bar{m}$ and $\bar{x}$, becomes

$$(\bar{m} + \bar{x})\frac{dv^2}{d\bar{x}} + v^2\left(1 + f\,\frac{L}{D}\,\bar{x}\right) = \frac{2P}{\rho}$$

from which it can be seen that for the same $\bar{m}$ and f the velocities of the projectiles in geometrically similar model and prototype are the same. In general, the same velocities will be obtained if

$$\bar{m}_m = \bar{m}_p \qquad f_m\bar{L}_m = f_p\bar{L}_p$$

If f is the same, and L/D is the same for model and prototype, $\bar{m}$ becomes the important parameter to be matched. For similitude under the action of inertia and gravity forces,

$$\frac{\text{Inertia force}}{\text{Gravity force}} = \frac{(m + \rho Ax)v\,\partial v/\partial x}{(m + \rho Ax)} \simeq \frac{v_T{}^2/D}{g}$$

so that if $v_T{}^2/Dg$ is matched between model and prototype, similitude of the two forces would be established. This, in fact, is the well-known Froude number. In most cases, since the duration of flight is small, the inertia and viscous forces become important for similitude considerations.

References

[1] Murray, J. J. and Kumar, S., "Vacuum-air missile boost," ARS J. **31**, 1443–1445 (October 1961).

[2] *1961 Missiles Volume* (Government Data Publications, Washington, D. C., 1961), pp. 92–99.

[3] "Nike X," Aviation Week Space Technol. **80**, 25 (February 3, 1964).

[4] "Mobile medium range ballistic missile (MMRBM)," Aviation Week Space Technol. **80**, 29 (March 2, 1964); also "Astrolog," Missiles and Rockets **14**, 25–31 (January 13, 1964).

[5] Perry, H. A., "Explosive bomb or weapon casing," U. S. Patent 2,977,885 (March 1955); also private communication (May 10, 1962).

[6] Foa, J. V., "Evaluation of the increase of missile payload that can be obtained from atmospheric boost at take off (project Mountainwell)," Army Research Office-Durham, TR (September 1961).

[7] Ordway, F. I., Gardner, J. P., Sharpe, M. R. Jr., Wakeford, R. C., *Applied Astronautics* (Prentice-Hall, Inc., Englewood Cliffs, N. J., 1963), Chap. 1, pp. 111–116.

[8] Ordway, F. I. and Wakeford, R. C., *International Missile and Spacecraft Guide* (McGraw-Hill Book Co., New York, 1960), pp. USA/55–66.

[9] Moody, L. F., "Friction factors for pipe flow," Trans. Am. Soc. Mech. Engrs. 671–684 (November 1944).

[10] Colebrook, C. F., "Turbulent flow in pipes, with particular reference to the transition region between the smooth and rough pipe laws," J. Inst. Civil Engrs. (London, England) **11**, 133–156 (1938–1939).

[11] Burr, J. W., Adair, W. R., and Madden, J. J., "Performance analysis of the Murray-Kumar missile boost system," TR AE G110, Dept. Aeronautical Engineering and Astronautics, Rennselaer Polytechnic Inst., Troy, N. Y. (December 1961).

[12] Prandtl, L. and Tietjens, O. G., *Applied Hydro and Aeromechanics* (Dover Publications, Inc., New York, 1957), Chap. II.